Combining Harmonic Functions and Random Sampling in Robot Motion Planning: A lazy approach

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Abstract

This paper presents a lazy procedure that enhances the performance of a robot motion planning method, called PHM, that uses a potential-field approach based on harmonic functions together with a random sampling scheme. The harmonic functions used to guide the solution path are computed over a 2^d-tree decomposition of a d-dimensional Configuration Space that is obtained with probabilistic cell sampling. This paper proposes a lazy variant of the PHM planner that eliminates, reduces or delays as much as possible any time-consuming computation. The proposed approach , therefore, makes the planner computationally more efficient.

1 Introduction

The planning of collision-free paths for a robot through the obstacles in a workspace is a difficult problem that is usually tackled in the robot's Configuration Space ($\mathcal{C}$-space). The dimension of the $\mathcal{C}$-space is generally high, since it is equal to the number of degrees of freedom of the robot, and the exact characterization of the obstacles in $\mathcal{C}$-space ($\mathcal{C}$-obstacles) is a barrier that precludes the use of many motion planning approaches. This is the reason why sampling-based motion planners, like Probabilistic Roadmap Methods (PRMs [1]) or those based on the Rapidly-exploring Random Trees (RRT [2]), are giving very good results in robot path planning problems with many degrees of freedom.

Other path planning approaches, like those based on potential-field methods have also given good results, although not in so high dimensional $\mathcal{C}$-spaces, since usually an approximate decomposition of $\mathcal{C}$-space is required [3]. Among them, those based on harmonic functions are interesting because they give rise to practical, resolution-complete planners without local minima [4].

The Probabilistic Harmonic-function based method (PHM) previously proposed by the authors [5], is an attempt to extend the use of potential-field based methods to higher dimensional $\mathcal{C}$-spaces. The basic idea is to combine them with a random sampling scheme (in a similar way as roadmap methods gave rise to PRM). The harmonic functions used to guide the solution path are computed over a 2^d-tree decomposition of a d-dimensional $\mathcal{C}$-space that is obtained with a probabilistic cell decomposition (cell sampling and classification). A similar approach in this line is performed in [6], where a collection of spherical balls of different radius covering the free $\mathcal{C}$-space is incrementally built following a sampling-based technique and used to compute a global navigation function.

The present paper is devoted to enhance the PHM planner following a lazy philosophy. Lazy evaluation approaches have already been used in PRM-like planners giving very good results [7] [8]. The present proposal is based on the dynamic combination of the computation of the harmonic functions and the search of the solution path that allows: a) to use a less expensive cell classification procedure; b) to perform a late cell evaluation; c) to use harmonic function values to bias cell sampling towards the more promising regions; and d) to use the minimum required resolution level. All these factors eliminate, reduce or delay time consuming computations as much as possible and, therefore, accelerate the finding of a solution path.

The paper is structured as follows. Section 2 reviews the basic PHM path planning method. Section 3 reviews the improvements proposed based on a lazy approach. Finally, Section 4 presents the modified PHM algorithm and Section 5 summarizes the contributions.

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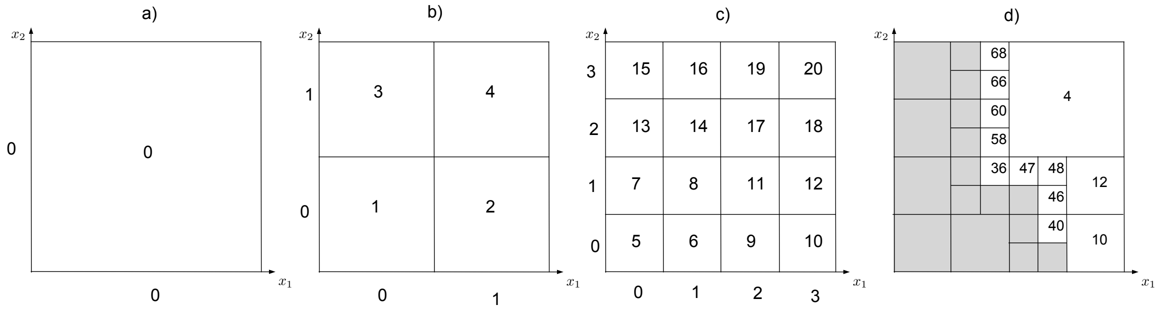

Figure 1: *Cell codes for different levels in the hierarchy in 2D $\mathcal{C}$-spaces: a) level 0, b) level 1, c) level 2; and d) An example of the codes of white cells of a quadtree.*

2 Basic PHM approach

2.1 $\mathcal{C}$-space decomposition

The following 2^d-tree decomposition of a d-dimensional $\mathcal{C}$-space is considered [9]. An initial cell, b^0, covering the entire $\mathcal{C}$-space is the tree root (b^0 is considered to have sides with unitary size). The levels in the tree are called partition levels and are enumerated such that the tree root is the partition level 0 and the maximum resolution corresponds to partion level M. Partition levels are denoted by super-indices: a cell of a given partition level m is called an m-cell, and denoted as b^m. The m-cells have sides of size $s_m = 1/2^m$.

A code convention that univocally labels and locates each cell of the 2^d-tree decomposition of $\mathcal{C}$-space is used. With this code convention, any subset of cells, e.g. those belonging to the subset $\mathcal{C}_{free}$ of collision-free configurations, can be managed as a list of codes, in a similar way as the *linear quadtrees* proposed in [10] for $d = 2$.

The cell codes are non-negative integers that univocally locate the cells in $\mathcal{C}$-space. The codes for a given partition level m range from C^m_{ini} to C^m_{end}, with:

$$C^m_{ini} = \frac{2^{dm} - 1}{2^d - 1} \quad (1)$$

$$C^m_{end} = 2^d C_{ini} \quad (2)$$

Since $C^m_{ini} = C^{(m-1)}_{end} + 1$, the proposed code convention uses all non-negative integers. Figure 1 shows the codes used for the cells of different partition levels for a 2D $\mathcal{C}$-space.

2.2 Harmonic Functions

An harmonic function ϕ on a domain $\Omega \subset \Re^n$ is a function that satisfies Laplace's equation:

$$\nabla^2 \phi = \sum_{i=1}^{n} \frac{\partial^2 \phi}{\partial x_i^2} = 0 \quad (3)$$

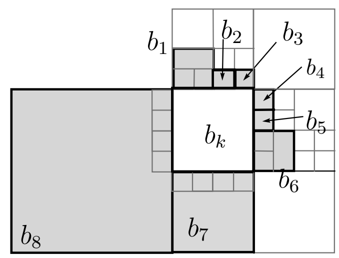

Figure 2: *Neighbor cells of different size used to compute the harmonic function at b_k. Cells b_2, b_3, b_4 and b_5 are M-cells.*

Harmonic functions are useful for motion planers based on potential-field methods since they do not have local minima [4]. The solution of the Laplace's equation is usually found numerically using finite difference methods, i.e. by sampling ϕ and its derivatives on a regular grid and using relaxation methods. Solutions over a non-regular grid like a 2^d-tree decomposition are also possible. Let b_k be a given cell, N be the actual number of neighbors of b_k, and W_j be the number of M-cells that can be contained in the border between b_k and a neighbor cell b_j. Then, the value u_k of the harmonic function, called the HF-value, at b_k is computed as the following weighted mean [9]:

$$u_k = \frac{\sum_{j=1}^{N} W_j u_j}{N_{max}} \quad (4)$$

As an example, in Figure 2 $N = 8$, $N_{max} = 16$ and:

$$u_k = \frac{2u_1 + u_2 + u_3 + u_4 + u_5 + 2u_6 + 4u_7 + 4u_8}{16} \quad (5)$$

Given a 2^d-tree decomposition of the $\mathcal{C}$-space where the neighboring relationship between cells is known, the relaxation methods are applied all over the free cells in a hierarchical top-down manner, i.e. the HF-values are consecutively computed from level 0 to level M in an iterative way until the convergence is attained. The obtained solution depends on the boundary conditions: the Dirichlet boundary

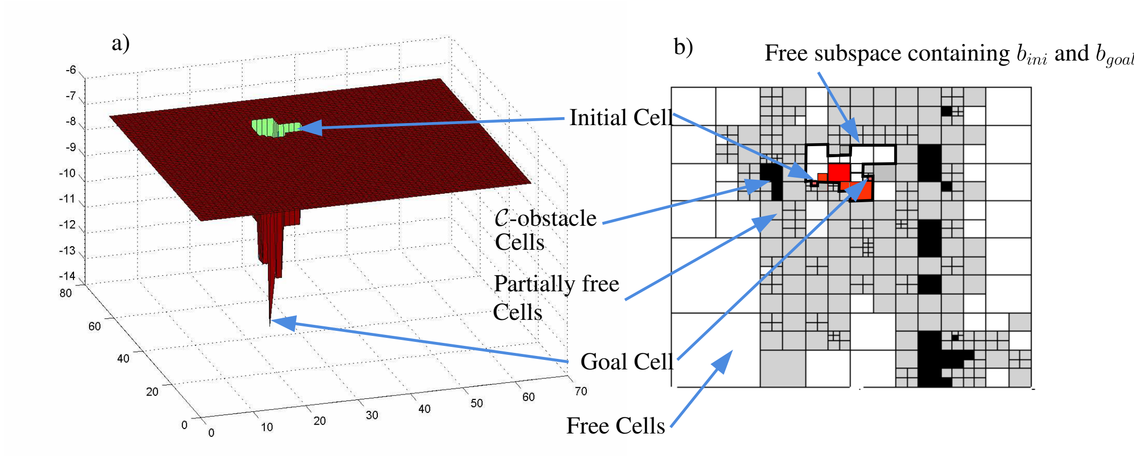

Figure 3: *a) Harmonic function computed over the free cells of a non-regular grid using the Dirichlet boundary condition (note that the final HF-value of the cells in free subspaces not containing b_{ini} and b_{goal} is equal to the fixed high HF-value of the obstacles); b) Solution path obtained following the negated gradient after having explored few cells.*

condition is used, which sets the $\mathcal{C}$-obstacle cells at a fixed high value and the goal cell at a low one. The initial value of u for the rest of the cells is set at an arbitrary middle value between both extremes.

2.3 PHM procedure

The basic PHM path planning algorithm was introduced in [9]. It combines a random cell sampling algorithm with the path planning performed using harmonic functions:

- Cell sampling: Cell sampling is done from the set G of partially free cells (gray). Initially G contains the cell b^0 covering the whole $\mathcal{C}$-space. Sampled cells are classified by a distance checker to be free cells (white), obstacle cells (black) or partially free cells. Free and obstacle cells are stored, respectively, into sets W and B; partially free cells are subdivided and stored in G to be explored later. Cells in G are of different size and sampling probability is set increasing with its volume. This leads to a rapid characterization of the $\mathcal{C}$-obstacles, since the uncertainty of big partially free cells is elucidated earlier.
- HF-Path planning: Harmonic function values are computed over the free cells, i.e. obstacle cells and partially free cells are considered as $\mathcal{C}$-obstacles. A channel is then searched from the initial cell b_{ini} to the goal cell b_{goal} following the negated gradient. Harmonic functions do have a unique minimum and therefore a solution path is found if it exists.
- PHM combination: The previous two steps are consecutively performed until a path is found or a maximum predefined number of cells has been explored, i.e if no path is found between b_{ini} and b_{goal} more samples are required and more cells must be sampled.

The PHM algorithm is as follows:

```
PHM-Path Planning(b_ini, b_goal)
  G ← b^0
  B ← ∅
  W ← ∅
  DO
    {G, W, B} ← Sample(G)
    H ← Compute Harmonic Function (W)
    p ← Find path(H, b_ini, b_goal)
    IF p ≠ ∅ RETURN p
  WHILE p = ∅ or n < N_max
  RETURN ∅
END
```

As an example, Figure 3 shows the harmonic function computed over the free cells of non-regular grid, that is used to find the solution path following the gradient descent. The PHM algorithm has been able to find the path having explored few of the cells of the $\mathcal{C}$-space.

3 Lazy PHM proposal

In order to accelerate the finding of a solution path, the following improvements are introduced to the basic PHM path planning algorithm that eliminate, reduce or delay as much as possible computations that are time consuming:

- In order to classify cells as free, obstacle or partially free cells without using costly distance computations, a probabilistic cell classification is proposed based on simple collision-check tests(Section 3.1).
- A late cell evaluation is proposed: solution paths with partially free cells are allowed, which delays collision-check tests until the end of the procedure (Section 3.2).
- Importance sampling is proposed in order to focus sampling in most promising regions and avoid sampling in areas that have no chance of containing a solution path (Section 3.3).
- A control of the level of resolution is proposed in order to find the solution using the minimum required resolution level (Section 3.4).

These proposals are detailed in the following subsections. Some of them are (partly) introduced in [5] and [11].

3.1 Probabilistic cell classification

The basic PHM approach relies on the availability of a distance checker able to provide both positive and negative (penetrating) distances in order to classify the cells. This requirement can be simplified to the availability of a collision checker if a probabilistic cell classification is performed, i.e. if a cell is classified as a function of the results of the collision checker at a discrete set of configurations of the cell.

The cell classification procedure is as follows. A given m-cell is classified depending on the result of a collision check performed at the configuration of its center and at the configurations of the centers of its subcells. These configurations are obtained by a deterministic sampling sequence that provide them in a low-dispersion order. The maximum number of configurations generated by the sequence to classify the cell is:

$$J = \sum_{i=m}^{M} 2^{d(i-m)} \tag{6}$$

that corresponds to the cardinality of the set composed of the given m-cell and all of its subcells, of levels ranging from $m+1$ to M.

Not all the J samples are needed to proceed with the cell classification. Instead, cell classification is performed with a subset $j < J$ of configurations, and updated each time new sample configurations are provided:

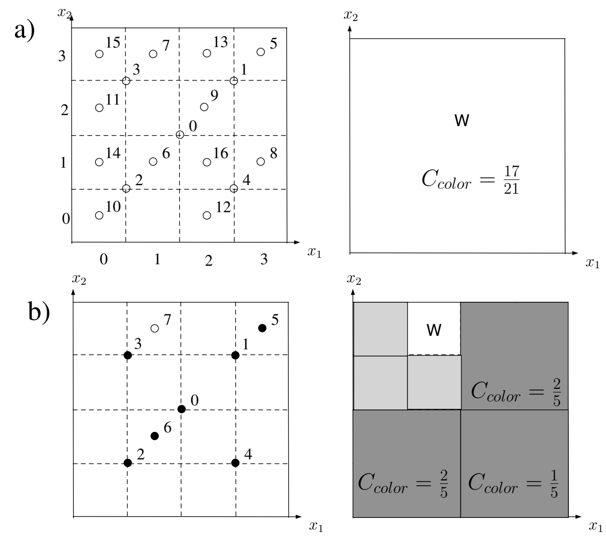

Figure 4: *Classification of a $(M-2)$-cell: a) the cell is classified as a free cell with $C_{color} = \frac{17}{21}$; b) the cell is subdivided into free, obstacle and unknown subcells.*

- If the result of the collision-check is the same for all the j configurations then the cell is classified as a free cell if the configurations are collision-free, or obstacle cell, otherwise. In these cases, the following degree of certainty, called color certainty, is assigned to the cell:

$$C_{color} = \frac{j}{J} \tag{7}$$

  As an example Figure 4a shows a cell classified as a free cell with $C_{color} = 17/21$.

- If the result of the collision-check is not the same for all the j configurations then the cell is subdivided as necessary until the obtained subcells contain only free configurations, or obstacle configurations, or do not contain any sample at all. In the first two cases these subcells are classified as free or obstacle cells (with their own degree of certainty), and those void subcells are classified as unknown subcells. As an example Figure 4c shows a $(M-2)$-cell where the first seven samples are classified as obstacle but the eighth (sample number 7) is classified as free. The cell is partitioned into: a white M-cell with $C_{color} = 1$, two black $(M-1)$-cells with $C_{color} = 2/5$, one black $(M-1)$-cell with $C_{color} = 1/5$, and three gray M-cells.

Cells classified as free are stored in W, those classified as obstacle are stored in B, and those classified as unknown are stored in G.

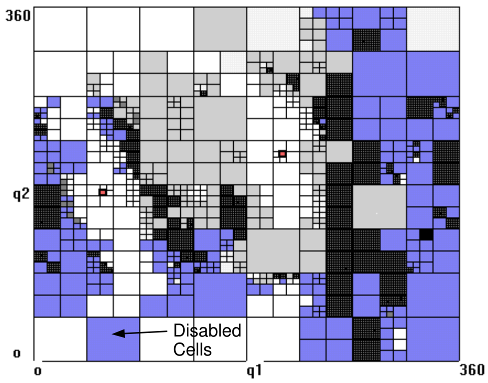

Figure 5: *Cell sampling disabled at cells that cannot be on the solution path because their HF-value is greater than the HF-value at* b_{ini}.

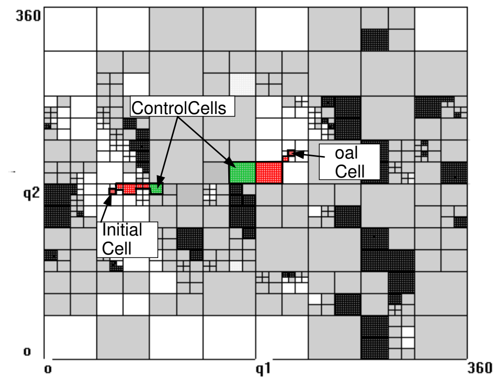

Figure 6: *The extremes of a broken path are labelled as control cells and used to compute an harmonic function that bias the sampling towards their environment.*

3.2 Late cell evaluation

The basic PHM approach computes the harmonic function over the white cells, which requires that a lot of cells be explored in order to be able to find a solution path. This can be relaxed by allowing the harmonic function to be computed over the white and gray cells. As a consequence, the solution path obtained must be verified to effectively be collision free, i.e. gray cells of the solution path must be verified to be white cells.

3.3 Importance sampling

In the basic PHM approach the cell sampling is biased only by the cell sizes (bigger cells have a greater sampling probability), i.e. the probability P to sample a cell b_k^m is weighted by 2^{-m}. The lazy PHM variant also weights the the probability P to sample a cell b_k by other two factors:

- *Color certainty*: weights the degree of knowledge of the occupancy of the cell. In the lazy PHM variant, not only the cells in G are to be sampled, but also those cells in B and W that do not have their color certainty equal to one. The less knowledge of a cell the higher its probability of being sampled (the weight is set to zero for the free and obstacles cells that have a color certainty equal to one, and to its maximum for the cells of G because they have color certainty equal to zero) [11].
- *HF-value*: weights the chance of a cell to be on the solution path, i.e. this factor makes P greater for cells near the goal cell (that have a lower harmonic function value) and makes P equal to zero for those cells that cannot be on the solution path (because the solution path is found following the negated gradient and their harmonic function value is higher than the corresponding to the initial cell - see the blue cells in Figure 5) [5].

This use of harmonic function values to bias the sampling can be further exploited if combined with the late cell evaluation commented in Section 3.2. If as a result of the evaluation of a path a set of few free non-connected segments is obtained, the extreme cells of those segments are labeled as control cells and used as follows. A secondary harmonic function is computed fixing the initial, the goal cell and all the control cells at a low potential. This secondary harmonic function is used to bias the probability of cell during the next cell sampling, i.e. cells near the control cells will have lower HF-value and hence a greater probability to be sampled. Figure 6 shows two free segments obtained after evaluating a solution path. Their extremes are labeled as control cells.

3.4 Resolution level

If a solution path exists with a clearance 2^{-c} then, this path can be found using a multi-grid with a maximum partition level $M = 2c$ (since the narrowest part of the solution channel will be composed of $2c$-cells, of size twice the clearance). Then, there is no point in sampling cells of the $\mathcal{C}$-space of partition levels $m > M$ because a path with maximum clearance is usually desired.

Therefore the lazy PHM variant is implemented with a variable maximum partition level M, i.e. initially the maximum partition level is fixed to an initial value M_0 and, if after some trials no path is found, M is increased and the path planning resumes.

4 Lazy PHM algorithm

The proposed improvements detailed in the previous section are incorporated into the lazy PHM algorithm, shown below, that uses the following functions:

- Compute HF: Computes the harmonic function over the white and gray cells. If control cells are passed as parameter then they are fixed at low potential to be used to bias the following cell sampling.
- Sample: samples a set of cells of $\mathcal{C}$-space with the sample probability P that considers the cell size, the color certainty and the HF-value, and classifies each cell into sets G, B and W (updating the color certainty and subdividing them if necessary).
- Find path: searches a path following the gradient descent and, if found, evaluates the cells that are not free (i.e. the gray cells and the white cells with $C_{color} < 1$). If a free path is found it is stored in p; otherwise p is set to null and then the control cells are stored in c.

```
PHM-Path Planning(b_ini, b_goal)
  G ← b^0
  B ← ∅
  W ← ∅
  M ← M_0
  DO
    {G, W, B} ← Sample({G, W, B}, H)
    H ← Compute HF (W, G)
    (p, c) ←Find path(H, b_ini, b_goal)
    IF p ≠ ∅ RETURN p
    IF c ≠ ∅ THEN H ← Compute HF (W, G, c)
    IF n > P_max THEN M ← M + ΔM
  WHILE p = ∅ or n < N_max
   RETURN ∅
END
```

5 Conclusions

Following a lazy philosophy this paper has introduced improvements to PHM, a path planning method based on the combination of a random sampling scheme (used to explore the $\mathcal{C}$-space) and a potential-field method based on harmonic functions (used to find a path following a gradient descent). The lazy PHM variant uses all the benefits of the combining of both methods. Besides the gradient descent guiding, harmonic function values are now also used to bias the random sampling, thus focusing the $\mathcal{C}$-space exploration towards the more promising regions. Also, collision-checks are delayed as much as possible by computing the harmonic function over the free and partially free cells, and cell evaluation is made probabilistic and based on collision-check tests instead of computationally expensive distance tests. Finally, the maximum grid resolution is now controlled allowing to find the solution using the minimum required resolution level. All these proposals eliminate, reduce or delay time consuming computations as much as possible and, therefore, make PHM computationally efficient.

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