

Finding Grasping Configurations of a Dexterous Hand and an Industrial Robot*

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Abstract—Given an industrial robot equipped with a dexterous hand and an object to be grasped with four grasping points determined on its faces, this paper deals with the problem of finding the joint configurations that allow to grasp that object. The proposed solution is based on an iterative optimization method that consecutively moves the joint that best contributes to reduce the distance of the fingertips to the desired locations. The method is particularized for a Stäubli RX90 robot and the dexterous hand MA-I with four fingers developed at the IOC's Robotics Lab.

Index Terms—Inverse kinematics, optimization methods, grasp planning.

I. INTRODUCTION

Dexterous hands are incorporated to robots in order to make them more flexible and widen the type of tasks they can perform. This fact involves many specific planning and control problems. The first problem to be tackled is the grasp synthesis that determines the best stable and proper grasp for a given object [1], i.e. where should the fingers be placed on the object and in which direction should the forces be exerted in order to grasp the object.

Once the grasping points on the object are known, the inverse kinematics of the hand-arm ensemble must be solved in order to determine the joint positions for the actual configuration of the object. This is a complex problem due to the great number of involved degrees of freedom and the tree structure of the kinematic chain.

This paper introduces a method to solve the inverse kinematic problem for any hand-arm ensemble described by its Denavit-Hartenberg parameters. The proposal is based on an optimization method that can cope with general robots and hands, although it is particularized for a Stäubli RX90 robot and the dexterous hand MA-I with four fingers developed at the IOC's Robotics Lab [2].

After this introduction, the paper is structured as follows. Section II describes the problem by presenting the kinematic model, the problem statement and the proposed solution based on an iterative optimization method. Section III presents the objective functions to be minimized, and Section IV describes the proposed inverse kinematics algorithm. The proposed method is validated with the experiments reported in Section V. Finally, Section VI presents the conclusions of the work.

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II. PROBLEM DESCRIPTION

A. Kinematic model

A robot arm equipped with a mechanical hand form a kinematic tree structure. Let A and F be the number of degrees of freedom of the arm and of each finger, respectively, and N be $N = A + F$. Let K be the number of fingers of the hand. The kinematic tree structure is considered as K kinematic chains that share the first A links. Then the links are labelled as j_k with $j = 1, \dots, N$ and $k = 1, \dots, K$. When the meaning is clear, the subindex will be omitted for the arm links, since $j_m = j_n \forall m, n \in \{1, \dots, K\}$ and $\forall j \in \{1, \dots, A\}$.

Using this nomenclature, let us define the following reference frames (Fig. 1):

- \mathcal{F}_W : world reference frame.
- \mathcal{F}_0 : reference frame attached to the base of the robot.
- \mathcal{F}_{j_k} : reference frame attached to link j_k . The frames attached to the fingertips are called \mathcal{F}_{N_k} , with $k = 1, \dots, K$.
- $\mathcal{F}_{N_k}^*$: desired position of \mathcal{F}_{N_k} .

The reference frames \mathcal{F}_0 , $\mathcal{F}_{N_k}^*$ and \mathcal{F}_{N_k} are described, with respect to frame \mathcal{F}_W , by homogeneous transformations T_W^0 , $T_W^{N_k^*}$ and $T_W^{N_k}$, respectively. Each reference frame \mathcal{F}_{j_k} is described with respect to the previous link reference frame, $\mathcal{F}_{(j-1)_k}$, by means of a homogeneous transformation, $T_{(j-1)_k}^{j_k}$. Using the Denavit-Hartenberg parameters α_{j_k} , θ_{j_k} , a_{j_k} and d_{j_k} , $T_{(j-1)_k}^{j_k}$ is given by [3]:

$$T_{(j-1)_k}^{j_k} = \begin{bmatrix} \cos \theta_{j_k} - \cos \alpha_{j_k} \sin \theta_{j_k} & \sin \alpha_{j_k} \sin \theta_{j_k} & a_{j_k} \cos \theta_{j_k} \\ \sin \theta_{j_k} & \cos \alpha_{j_k} \cos \theta_{j_k} & a_{j_k} \sin \theta_{j_k} \\ 0 & \sin \theta_{j_k} & d_{j_k} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

B. Particular hand-arm ensemble

Fig. 2 show the CAD models of the hand MA-I and the robot Stäubli RX90 as well as the real hand-arm ensemble.

The dexterous hand MA-I developed at the IOC's Robotics Lab has four fingers with four degrees of freedom each one. Three additional virtual joints are considered at the fingertips in order to take into consideration that (Fig. 3):

- Any point of the fingertip can be used to contact with the selected grasping point on the object. Assuming a spherical fingertip, the contact fingertip point can be determined by two virtual joints, θ_{11_k} and θ_{12_k} , and a virtual link of length the radius of the sphere. The range of these virtual joints determines the valid area of the fingertip sphere.

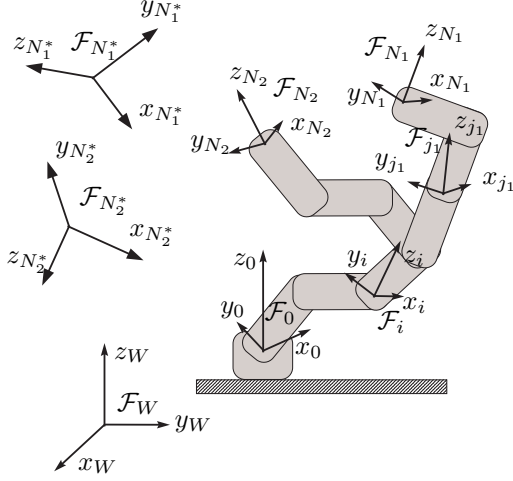


Fig. 1. Definition of reference frames.

- Only the direction normal to the object surface at the grasping contact point is defined as a requirement for the grasping and, therefore, the orientation of the finger around this normal is free. This is modelled by another virtual joint, θ_{13k} , with a range of 2π .

The DH parameters of the hand and the arm, including the three virtual joints of the fingertips, are shown in the Appendix.

C. Problem statement and proposed solution

The problem to be solved is “which are the proper positions of the arm and hand joints in order to perform a given grasp?”, i.e. find the set of joint values that locate the fingertip reference frames \mathcal{F}_{N_k} at given desired locations $\mathcal{F}_{N_k}^*$ with $k = 1, \dots, K$.

The proposed solution to this inverse kinematics problem is based on an iterative optimization method. The objective function to be optimized (minimized) is the distance from the current to the desired fingertip locations. The problem is decoupled by analyzing the effect of the individual motion of each joint in this objective function, i.e. the problem is partitioned into several one-degree of freedom optimization subproblems. At each step of the iterative process, the joint value that individually minimize this objective function is computed and used to update the kinematic structure.

The proposed solution is inspired by the Distributed Optimization Method introduced by Regnier et al. [4] to solve the inverse kinematics of all serial manipulators, and also used to solve the problem of the kinematic synthesis of manipulators [5]. A similar approach, but using a different distance metrics, was presented by Ahuactzin and Gupta [6] to solve the inverse kinematics problem for redundant manipulators. The solution proposed in this paper uses a different distance metrics and extends the approach to kinematic-tree structures.

III. OBJECTIVE FUNCTIONS

A. Distance metrics

The representation of the distance between two reference frames involves the parameterized mix of translational

and rotational components. No bi-invariant metrics exist in $SE(3)$, the Euclidean group of rigid-body motions, although left- or right-invariant distance metrics can be proposed [7] (i.e. distances invariant with respect to the choice of the inertial frame or to the choice of the rigid body frame, respectively). The use of these metrics can be computationally expensive and therefore simpler metrics are usually proposed in iterative procedures (e.g. [4], [6]). These simpler metrics may not have such invariant features nor a well-defined mix of translational and rotational components. These factors may influence the behavior of metric-based complex algorithms in an unclear and, therefore, undesirable way. As a consequence, the left-invariant metric proposed in [7] is used in this paper.

Let X_1 and X_2 be two homogeneous transformations defining two reference frames. Then the distance between them is determined by:

$$dist(X_1, X_2) = \sqrt{\phi^2 + \frac{1}{L^2}\Delta^2} \quad (2)$$

where ϕ is the angle, around a given axis, that X_1 must rotate in order to obtain the same orientation as X_2 , Δ is the euclidian distance between the origins, and L is a parameter that weights the translational and rotational components.

The translational distance Δ and the rotational distance ϕ are computed in the following subsections considering the kinematic chain formed by the robot and one finger assuming $\mathcal{F}_W = \mathcal{F}_0$. In this case $\mathcal{F}_{N_k}^*$ and \mathcal{F}_{N_k} are described, respectively, by $T_0^{N_k^*}$ and $T_0^{N_k}$. Therefore:

$$dist(X_1, X_2) = dist(T_0^{N_k}, T_0^{N_k^*}) \quad (3)$$

Since this is a left invariant distance, this equation can be reformulated as (Fig. 4):

$$dist(X_1, X_2) = dist(T_{(j-1)k}^{jk} \cdot T_{jk}^{N_k}, (T_0^{(j-1)k})^{-1} \cdot T_0^{N_k^*}) \quad (4)$$

This equation is written considering θ_{jk} as the unique variable (i.e. considering fixed all other joint values). This allows to find the value of θ_{jk} that minimizes the distance. Let the translational and rotational components be called Δ_{jk} and ϕ_{jk} , respectively. Their expressions are the followings:

1) *Translational distance:* Let the homogeneous transformations $T_{jk}^{N_k}$ and $\left[(T_0^{(j-1)k})^{-1} \cdot T_0^{N_k^*} \right]$ be expressed as:

$$T_{jk}^{N_k} = \begin{bmatrix} t_{00} & t_{01} & t_{02} & t_{03} \\ t_{10} & t_{11} & t_{12} & t_{13} \\ t_{20} & t_{21} & t_{22} & t_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$\left[(T_0^{(j-1)k})^{-1} \cdot T_0^{N_k^*} \right] = \begin{bmatrix} h_{00} & H_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

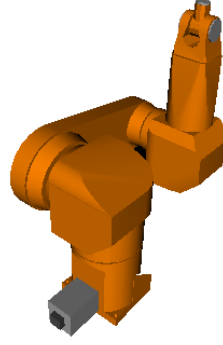
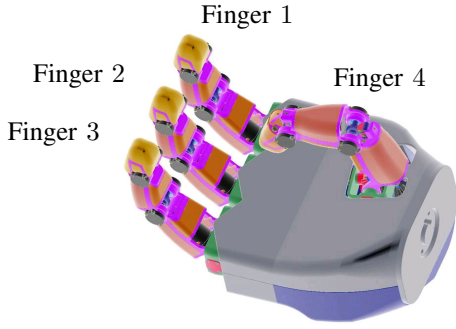


Fig. 2. CAD models of the dexterous hand MA-I and the Stäubli RX90 robot and the physical hand-arm ensemble.

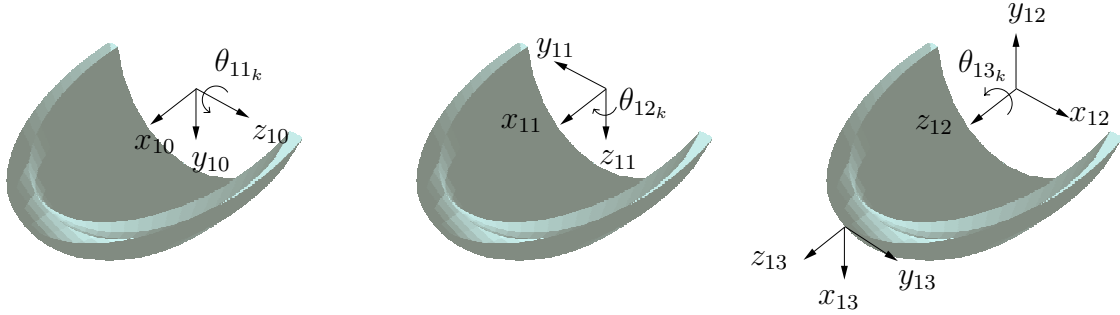


Fig. 3. Fingertip with the virtual revolute joints θ_{11_k} , θ_{12_k} and θ_{13_k} defined around the z -axis of \mathcal{F}_{10_k} , \mathcal{F}_{11_k} , \mathcal{F}_{12_k} , respectively.

Then, being $T_{(j-1)_k}^{j_k}$ defined by (1), the square of the translational distance Δ_{j_k} is:

$$\begin{aligned} \Delta_{j_k}^2 &= P_{j_k} \cos(\theta_{j_k}) + Q_{j_k} \sin(\theta_{j_k}) + R_{j_k} \\ P_{j_k} &= 2(-t_{03}h_{03} - \cos \alpha_{j_k} t_{13}h_{13} + \sin \alpha_{j_k} t_{23}h_{13} - a_{j_k} h_{03}) \\ Q_{j_k} &= 2(-t_{03}h_{13} - \cos \alpha_{j_k} t_{13}h_{03} - \sin \alpha_{j_k} t_{23}h_{03} - a_{j_k} h_{13}) \\ R_{j_k} &= 2 \cos \alpha_{j_k} (t_{23}d_{j_k} - t_{23}h_{23}) + 2 \sin \alpha_{j_k} (t_{13}d_{j_k} - t_{13}h_{23}) \\ &\quad + h_{03}^2 + t_{13}^2 + t_{23}^2 + t_{03}^2 + a_{j_k}^2 + h_{13}^2 \\ &\quad + d_{j_k}^2 - 2d_{j_k}h_{23} + h_{23}^2 + 2t_{03}a_{j_k} \end{aligned} \quad (7)$$

2) *Rotational distance*: If Φ_1 and Φ_2 are the rotational matrices associated to $\left[\left(T_0^{(j-1)_k} \right)^{-1} \cdot T_0^{N_k^*} \right]$ and $\left[T_{(j-1)_k}^{j_k} \cdot T_{j_k}^{N_k} \right]$, respectively, then [7]:

$$\phi_{j_k}^2 = \arccos^2 \left(\frac{\text{tr}(\Phi_1^{-1} \Phi_2) - 1}{2} \right) \quad (8)$$

where $\text{tr}(\cdot)$ means the trace of a matrix.

This expression can be approximated by the following one:

$$\phi_{j_k}^2 = \frac{\pi^2}{2} \left(1 - \frac{\text{tr}(\Phi_1^{-1} \Phi_2) - 1}{2} \right) \quad (9)$$

If Φ_1^{-1} is expressed as:

$$\Phi_1^{-1} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \quad (10)$$

Then, the square of the rotational distance ϕ_{j_k} is:

$$\begin{aligned} \phi_{j_k}^2 &= \frac{\pi^2}{4} (3 - A_{j_k} \cos(\theta_{j_k}) - B_{j_k} \sin(\theta_{j_k}) - C_{j_k}) \\ A_{j_k} &= a_{00}t_{00} + a_{10}t_{01} + a_{20}t_{02} + \\ &\quad \cos \alpha_{j_k} (a_{01}t_{10} + a_{11}t_{11} + a_{21}t_{12}) - \\ &\quad - \sin \alpha_{j_k} (a_{01}t_{20} + a_{11}t_{21} + a_{21}t_{22}) \\ B_{j_k} &= a_{01}t_{00} + a_{11}t_{01} + a_{21}t_{02} - \\ &\quad \cos \alpha_{j_k} (a_{00}t_{10} + a_{10}t_{11} + a_{20}t_{12}) + \\ &\quad + \sin \alpha_{j_k} (a_{00}t_{20} + a_{10}t_{21} + a_{20}t_{22}) \\ C_{j_k} &= \cos \alpha_{j_k} (a_{02}t_{20} + a_{12}t_{21} + a_{22}t_{22}) + \\ &\quad \sin \alpha_{j_k} (a_{02}t_{10} + a_{12}t_{11} + a_{22}t_{12}) \end{aligned} \quad (11)$$

B. Objective functions for finger joints

The inverse kinematics problem can be reformulated as a set of one degree of freedom optimization subproblems, with the following objective functions to be minimized:

$$F_{j_k} = \text{dist}(T_{(j-1)_k}^{j_k} \cdot T_{j_k}^{N_k}, \left(T_0^{(j-1)_k} \right)^{-1} \cdot T_0^{N_k^*}) \quad (12)$$

i.e. the distance from the current to the desired location of the fingertip reference frame is rewritten as a function of each joint θ_{j_k} with $j = (A+1), \dots, (A+F)$, $k = 1, \dots, K$. Taking into account (2),(7) and (11), the resulting objective function is:

$$F_{j_k} = \frac{\pi^2}{4} (3 - A_{j_k} \cos \theta_{j_k} - B_{j_k} \sin \theta_{j_k} - C_{j_k}) + \frac{1}{L^2} (P_{j_k} \cos \theta_{j_k} + Q_{j_k} \sin \theta_{j_k} + R_{j_k}) \quad (13)$$

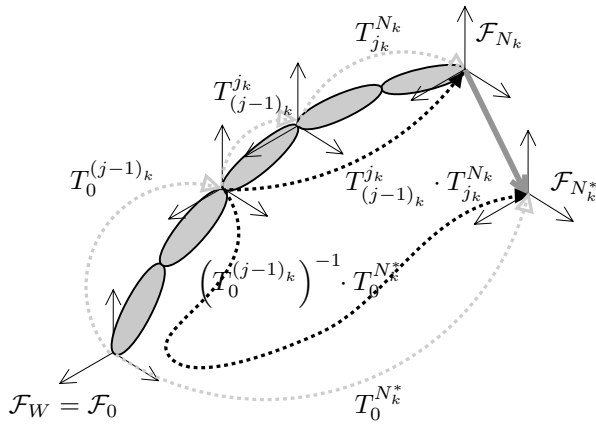


Fig. 4. Distance from the current to the desired reference frame of the fingertip k , measured with respect to reference frame of link j_k (using $T_W^0 = I$).

This expression can be rewritten as:

$$\begin{aligned}
 F_{j_k} &= \alpha_{j_k} \cos \theta_{j_k} + \beta_{j_k} \sin \theta_{j_k} + \gamma_{j_k} \\
 \alpha_{j_k} &= \frac{1}{L^2} P_{j_k} - \frac{\pi^2}{4} A_{j_k} \\
 \beta_{j_k} &= \frac{1}{L^2} Q_{j_k} - \frac{\pi^2}{4} B_{j_k} \\
 \gamma_{j_k} &= \frac{\pi^2}{4} (3 - C_{j_k}) + \frac{1}{L^2} R_{j_k}
 \end{aligned} \quad (14)$$

C. Objective functions for arm joints

The motion of the arm joints affect the position and orientation of the reference frames of all the fingers. Therefore, a different objective function for these joints must be defined in order to take into account this fact.

Let F_j^k be the objective function that gives the distance from the current to the desired position of the fingertip k , measured in the reference frame of the arm link j , with $j = 1, \dots, A$.

The function F_j^k is expressed by (14), since a unique kinematic serial chain is considered. Then, in order to consider all the kinematic chains, the objective function F_j of the arm joint θ_j is computed as the sum of the functions F_j^k , with $k \in 1 \dots K$:

$$F_j = \sum_{k=1}^K F_j^k = \sum_{k=1}^K \alpha_{j_k} \cos \theta_{j_k} + \beta_{j_k} \sin \theta_{j_k} + \gamma_{j_k} \quad (15)$$

Using the DH notation, the z axis of the reference frame \mathcal{F}_j is set coincident with the axis of the joint θ_{j+1} . This poses a problem for the last link of the arm, since with this convention K reference frames are defined at link A and therefore K measures of joint θ_A are obtained, although they differ only by a constant offset value [8]. In the proposed approach, the correspondence between these values is arbitrarily set with respect to finger 1:

$$\theta_{A_k} = \theta_{A_1} + \delta_{A_k} \quad k = 1 \dots K \quad (16)$$

with $\delta_{A_1} = 0$ and δ_{A_k} dependant on the geometry. For the hand MA-I these offset values are $\delta_{A_2} = \delta_{A_3} = 0$ and $\delta_{A_4} = 33.7^\circ$.

In order to take into account this fact, (15) has to be modified for joint θ_A as follows:

$$F_A = \sum_{k=1}^K \alpha_{A_k} \cos(\theta_{A_1} + \delta_{A_k}) + \beta_{A_k} \sin(\theta_{A_1} + \delta_{A_k}) + \gamma_{A_k} \quad (17)$$

D. Minimization of the objective function

The objective functions presented in the previous sections measure the distance from the current to the desired location of the fingertips, as a function of each single joint value. Then, these objective functions can be minimized to obtain the optimum joint values, i.e. the values of the joints that move the fingertips close to their desired location.

The joint value that minimizes F_{j_k} is obtained from $\frac{\partial F_{j_k}}{\partial \theta_{j_k}} = 0$ using:

- equation (15) for links $j = 1, \dots, (A - 1)$
- equation (17) for link $j = A$
- equation (14) for links j_k with $k = 1, \dots, K$ and $j = (A + 1), \dots, (A + F)$.

The value θ_{j_k} obtained is:

$$\theta_{j_k} = \begin{cases} \arctan \frac{\sum_{k=1}^K \alpha_{j_k}}{\sum_{k=1}^K \beta_{j_k}} & \text{when } j = 1, \dots, (A - 1) \\ \arctan \frac{\sum_{k=1}^K \{\alpha_{A_k} \cos \delta_{A_k} + \beta_{A_k} \sin \delta_{A_k}\}}{\sum_{k=1}^K \{-\alpha_{A_k} \sin \delta_{A_k} + \beta_{A_k} \cos \delta_{A_k}\}} & \text{when } j = A \\ \arctan \frac{\alpha_{j_k}}{\beta_{j_k}} & \text{when } j = (A + 1), \dots, (A + F) \end{cases} \quad k = 1, \dots, K \quad (18)$$

The value of θ_{j_k} is checked to be a minimum by verifying that the sign of the second derivative is positive. If this is not the case, the minimum occurs at $(\theta_{j_k} + \pi)$.

When θ_{j_k} is outside the range $[\theta_{j_k}^{min}, \theta_{j_k}^{max}]$ of possible joint values, it is set to the limit value:

$$\text{if } \theta_{j_k} < \theta_{j_k}^{min} \quad \text{then } \theta_{j_k} = \theta_{j_k}^{min} \quad (19)$$

$$\text{if } \theta_{j_k} > \theta_{j_k}^{max} \quad \text{then } \theta_{j_k} = \theta_{j_k}^{max} \quad (20)$$

IV. OPTIMIZATION METHOD

The inverse kinematics of the hand-arm ensemble is solved with an optimization method based on a procedure that iteratively computes the objective functions and moves the joint that best approaches the hand to the desired configuration.

The success of the proposed method depends on the initial joint values, due to the nature of iterative optimization algorithms. When the improvement of the iterative procedure is not good enough (measured as a relative decrement of the objective function), a retrial is performed restarting the procedure from a new initial configuration. A deterministic sampling sequence is used to uniformly generate initial configurations over the configuration space.

A. Hand-arm inverse kinematics algorithm

The algorithm to compute the hand-arm inverse kinematics, shown below, uses the following functions:

OptimizeArm: This function uses (18) for $j = 1, \dots, A$ to find the value of the arm joint that minimizes the mean distance from \mathcal{F}_{N_k} to $\mathcal{F}_{N_k^*}$ with $k = 1, \dots, K$. The function returns this joint value.

OptimizeFinger: For a given finger k , this function uses (18) for $j = (A + 1), \dots, (A + F)$ to find the value of the finger joint that minimizes the distance from \mathcal{F}_{N_k} to $\mathcal{F}_{N_k^*}$. The function returns:

- this optimum joint value.
- the value of the objective function, \mathbf{F}_k

MoveJoint: This function moves a specified joint, j_k , to the given value, θ_{j_k} .

StartConfiguration: This function generates the initial values of the arm joints using a deterministic sampling sequence. The initial finger joint values are set to the middle value of the corresponding ranges. The function returns a vector Θ^{ini} with those values.

Hand-Arm Inverse Kinematics($\mathcal{F}_{N_1^*}, \dots, \mathcal{F}_{N_K^*}$)

```

 $I_{max}$  = Maximum number of iterations
 $i = 0$ 
DO
   $retry = 0$ 
   $\Theta^{ini} = \text{StartConfiguration}()$ 
  DO
     $\theta_j = \text{OptimizeArm}(\mathcal{F}_{N_1^*}, \dots, \mathcal{F}_{N_K^*}, \Theta^{ini})$ 
     $\text{MoveJoint}(\theta_j)$ 
    FOR  $k = 1$  TO  $K$  DO
       $(\theta_{j_k}, \mathbf{F}_k) = \text{OptimizeFinger}(\mathcal{F}_{N_k^*})$ 
       $\text{MoveJoint}(\theta_{j_k})$ 
    END FOR
     $\mathbf{F} = \frac{1}{K} \sum_{i=1}^K \mathbf{F}_i$ 
    IF  $i \bmod 10$  THEN
      IF  $(\mathbf{F}_{ant} - \mathbf{F}) / \mathbf{F} < \delta$  THEN  $retry = 1$ 
       $\mathbf{F}_{ant} = \mathbf{F}$ 
    END IF
    IF  $\mathbf{F} < \epsilon$  RETURN  $(\theta_1, \dots, \theta_{N_K})$ 
     $i = i + 1$ 
  WHILE  $i < I_{max}$  AND  $retry = 0$ 
  WHILE  $i < I_{max}$ 
  RETURN solution not found
END

```

B. Initial configurations

The initial configurations of the arm joints are determined by sampling the corresponding configuration space. This problem is the same as the one encountered in sampling-based motion planners. Usually these planners randomly sample the configuration space (often with an heuristic bias towards the regions where it is most difficult to find a path). However, other sampling-based motion planners rely on deterministic sampling sequences. Deterministic sampling provide a good uniform and incremental coverage of the space, and can outperform random

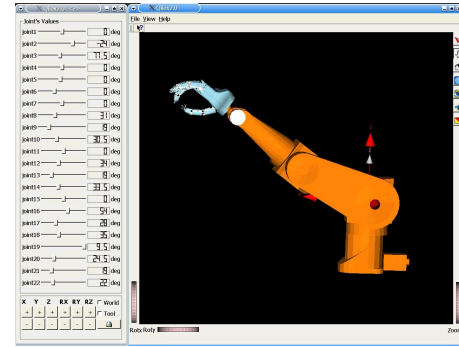


Fig. 5. Cell simulator with the RX90 robot and the MA-I hand.

sampling in nearly all motion planning problems [9]. The determination of the initial configurations of the proposed optimization method has been done using both random and deterministic sampling [10]. The best results were obtained using deterministic sampling.

V. VALIDATION

The inverse kinematics algorithm has been incorporated to the Qilex robotics simulator developed at the IOC's Robotic Lab (<http://qilex.berlios.de/>, Fig. 5), and has been statistically tested. The validation consisted of:

- Generating a set of grasping configurations by randomly setting the values of all the joints of the hand-arm ensemble and then computing the direct kinematics.
- Applying the inverse kinematics algorithm to the test set.

The test set is composed of 1,000 grasping configurations. The algorithm has been able to find a solution of the inverse kinematics in the 100% of the cases, using $I_{max} = 25,000$, $\epsilon = 0.00001$ and $\delta = 0.01$. The value that weighs the translational distance is fixed at $L = 80$. The mean number of iterations was 3,997 and the mean number of retrials was 18. The histograms of the corresponding results for the 1,000 test configurations are shown in Figs 6 and 7.

These values are drastically reduced when the test configurations are consecutive configurations of a path close to each other, and the joint values of one configuration are used as the initial ones for the next. In this case, experiments using real paths reported a mean of 88 ms to find the inverse kinematic for each configuration, always finding the solution within the first trial and using less than 500 iterations. Other tests with non-reachable grasping configurations were carried out, and the algorithm correctly reported "solution not found" after I_{max} trials.

VI. CONCLUSIONS

The use of dexterous hands in industrial robots pose several difficult problems, one of them being the determination of the arm and finger joints in order to grasp a given object (once the grasping points on its surface and the direction of the forces to be exerted are known). An iterative optimization method has been proposed to solve the inverse kinematics problem of any hand-arm ensemble described

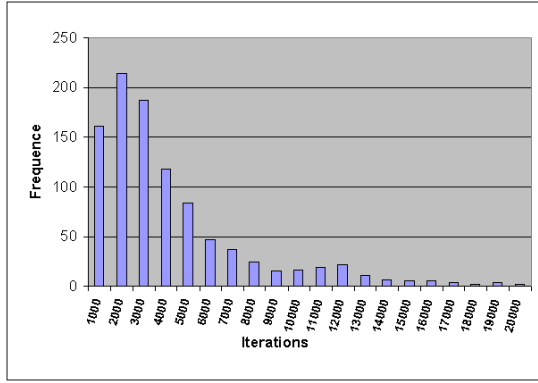


Fig. 6. Histogram of number of iterations.

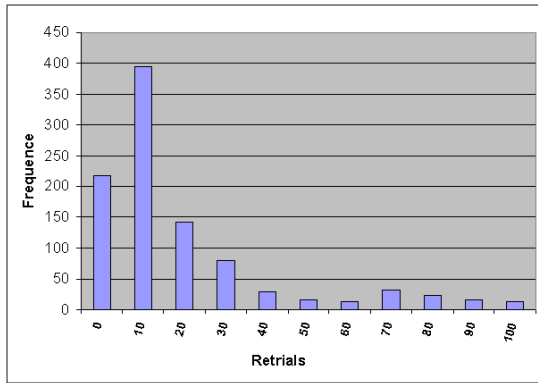


Fig. 7. Histogram of number of retrials.

by its DH parameters. The method consecutively finds the joint motions that best contributes to reach the goal. The objective functions to be minimized are the distances from the fingertips to the grasping points. Distance metrics has been carefully handled in order to properly consider orientations. The solution has been particularized for a Stäubli RX90 robot and the dexterous hand MA-I with four fingers developed at the IOC's Robotics Lab. The approach has been validated through exhaustive experiments on a simulator.

APPENDIX

The Denavit-Hartenberg parameters of the fingers of MA-I and the robot Stäubli RX90 are detailed in the the following tables (angles expressed in degrees and distances in millimeters). Note that the parameters of the last joint of the robot, θ_6 , are expressed in each finger table with different values due to the feature of the DH notation commented in Section III-C.

RX90 Stäubli robot (joints $j_k, j = 1, \dots, 5; \forall k$)

| | | | | | |
|----------------------|------|--------|-------|------|------|
| j_k | 1 | 2 | 3 | 4 | 5 |
| α_{j_k} | -90 | 0 | 90 | -90 | 90 |
| a_{j_k} | 0 | 450 | 0 | 0 | 0 |
| d_{j_k} | 0 | 0 | 0 | 450 | 0 |
| $\theta_{j_k}^{min}$ | -160 | -227.5 | -52.5 | -270 | -105 |
| $\theta_{j_k}^{max}$ | 160 | 47.5 | 232.5 | 270 | 120 |

Finger 1 (joints $j_k, j = 6, \dots, 13; k = 1$)

| | | | | | | | | |
|----------------------|------|-----|----|----|----|-----|------|------|
| j_k | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| α_{j_k} | 90 | 90 | 0 | 0 | 0 | -90 | -90 | 0 |
| a_{j_k} | 67 | 0 | 76 | 56 | 40 | 0 | 0 | 0 |
| d_{j_k} | 276 | 11 | 0 | 0 | 0 | 0 | 0 | 15 |
| $\theta_{j_k}^{min}$ | -180 | 80 | 0 | 0 | 0 | 0 | -180 | -270 |
| $\theta_{j_k}^{max}$ | 360 | 100 | 90 | 90 | 90 | 90 | 0 | 90 |

Finger 2 (joints $j_k, j = 6, \dots, 13; k = 2$)

| | | | | | | | | |
|----------------------|------|-----|----|----|----|-----|------|------|
| j_k | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| α_{j_k} | 90 | 90 | 0 | 0 | 0 | -90 | -90 | 0 |
| a_{j_k} | 0 | 0 | 0 | 56 | 40 | 0 | 0 | 0 |
| d_{j_k} | 276 | 11 | 0 | 0 | 0 | 0 | 0 | 15 |
| $\theta_{j_k}^{min}$ | -180 | 80 | 0 | 0 | 0 | 0 | -180 | -270 |
| $\theta_{j_k}^{max}$ | 360 | 100 | 90 | 90 | 90 | 90 | 0 | 90 |

Finger 3 (joints $j_k, j = 6, \dots, 13; k = 3$)

| | | | | | | | | |
|----------------------|------|-----|----|----|----|-----|------|------|
| j_k | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| α_{j_k} | 90 | 90 | 0 | 0 | 0 | -90 | -90 | 0 |
| a_{j_k} | -67 | 0 | 76 | 56 | 40 | 0 | 0 | 0 |
| d_{j_k} | 276 | 11 | 0 | 0 | 0 | 0 | 0 | 15 |
| $\theta_{j_k}^{min}$ | -180 | 80 | 0 | 0 | 0 | 0 | -180 | -270 |
| $\theta_{j_k}^{max}$ | 360 | 100 | 90 | 90 | 90 | 90 | 0 | 90 |

Finger 4 (joints $j_k, j = 6, \dots, 13; k = 4$)

| | | | | | | | | |
|----------------------|--------|-------|----|----|----|-----|------|------|
| j_k | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| α_{j_k} | 0 | 90 | 0 | 0 | 0 | -90 | -90 | 0 |
| a_{j_k} | 72 | 0 | 76 | 66 | 45 | 0 | 0 | 0 |
| d_{j_k} | 145 | 0 | 0 | 0 | 0 | 0 | 0 | 15 |
| $\theta_{j_k}^{min}$ | -213.7 | -66.3 | 0 | 0 | 0 | 0 | -180 | -270 |
| $\theta_{j_k}^{max}$ | 326.3 | -46.3 | 90 | 90 | 90 | 90 | 0 | 90 |

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